To add or to multiply?

An investigation of the role of preference in children's solutions of word problems

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Abstract

Previous research has shown that upper primary school children frequently erroneously solve additive word problems multiplicatively, while younger children frequently erroneously solve multiplicative word problems additively. It has been suggested that children’s preference for additive or multiplicative relations explains these errors, besides their lacking skills, but this claim has not been tested empirically yet. Therefore, we administered four test instruments (a word problem test, a preference test, and two tests measuring additive and multiplicative computation and discrimination skill) to 246 third to sixth graders. Previous research results on errors in word problems, as well as on preference were replicated and systematized. Further, they were extended by explaining this erroneous word problem solving behavior by preference, for those children who unmistakably had acquired the necessary computation and discrimination skills. This finding provides strong evidence for the unique additional role of children’s preference in erroneous additive or multiplicative word problem solving behavior.

Key words: word problem solving, additive reasoning, multiplicative reasoning, preference, skill

1. Introduction

Multiplicative reasoning is omnipresent in daily life situations: converting currencies from € to $, cooking from a cookbook recipe for four people for a company of eight, making scale drawings of buildings, using the constant driving speed to estimate the time of a journey, calculating the total price of a set of products based on the unit price, etc. Multiplicative reasoning is "the capstone of children's primary school arithmetic, and the cornerstone of all that is to follow" (Lesh, Post, & Behr, 1988, p. 94). Hence, it is crucial for understanding many mathematical ideas, such as fractions, functions, probability, statistics, and algebra (Lamon, 1993; Lamon & Lesh, 1992; Lesh et al., 1988; Vergnaud, 1988).

Learning to reason multiplicatively is a major goal in upper primary mathematics education. A typical way to practice it is through multiplicative missing-value word problems – also called rule-of-three problems by Vergnaud (1983, 1988) – wherein three quantities are given and the goal is to find the fourth one by identifying the multiplicative relation between two given quantities and applying this relation to the third given quantity (Cramer & Post, 1993; Harel and Behr, 1989; Kaput & West, 1994; Tourniaire and Pulos, 1985). For example, the missing value in the following problem of Kaput and West (1994) "A car of the future will be able to travel 8 miles in 2 minutes. How far will it travel in 6 minutes?" can be obtained by assuming a multiplicative relation among quantities of the same nature (also called the internal ratio, i.e., if the time is tripled, the distance is tripled; 2×3=6, so 8×3=24), or among quantities of a different nature (also called the external ratio, i.e., multiplying the time by four to obtain the distance; 2×4=8, so 6×4=24) (Karplus, Pulos, & Stage, 1983; Noelting, 1980; Vergnaud, 1983). A simpler way to solve this problem multiplicatively is by using a building-up strategy (Kaput & West, 1994), which does not rely on the operations of multiplication or division but on repeated addition or subtraction (e.g., 2+2+2 minutes to cover 8+8+8 miles). It remains controversial whether a building-up strategy is a valuable stepping stone in the development of multiplicative reasoning (e.g., Greer, 1992; Verschaffel, Greer, & De Corte, 2007). While some argue that repeated addition is an "implicit, unconscious and primitive intuitive model" for the arithmetical operation of multiplication (Fischbein, Deri, Nello, & Marino, 1985, p. 4), others stress that the links between addition and multiplication and between subtraction and division are merely procedural (Nunes & Bryant, 2010; Thompson & Saldanha, 2003). We consider the building-up strategy as an adequate multiplicative strategy in multiplicative missing-value word problems: It may be procedurally not advanced, but it reflects a correct consideration of the multiplicative relations between the quantities in the problem.

However, not all word problems wherein three quantities are given and the goal is to find the fourth one by identifying the relation between two quantities and applying this to the third quantity, must be solved multiplicatively; some need an additive solution. Take, for instance, the following missing-value word problem of Cramer, Post and Currier (1993): "Sue and Julie were running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?" (p. 159): Sue is six laps ahead of Julie the first time, and this difference will stay the same, since they are running equally fast (i.e., 3+6=9, so 15+6=21).

Both the car and runner problem contain three values and a fourth missing one, and these are linked in a specific way: The two quantities in the problem *co-vary* (Lamon, 2008). The more time has passed, the more distance the car has covered. Likewise, the more laps Julie has run, the more laps Sue has run. The relation between the two quantities remains *invariant*, irrespective of their exact values (Behr & Harel, 1990; Lamon, 2008). No matter how many laps Sue has run, she will always stay the same number of laps ahead of Julie; and irrespective of the distance that the car has already driven, it will still cover the same number of miles per minute. The relational reasoning required in missing-value problems involves considering the relation between two values and applying this relation to two other values.

There is, however, also an important difference between the two word problems that relates to the *nature* of the invariance. In the car problem, the quantities are linked by multiplication and division and the *ratio* between quantities is invariant (i.e., "multiplicative invariance", Behr & Harel, 1990), whereas in the runner problem the quantities are linked additively and the *difference* between quantities is invariant (i.e., "additive invariance", Behr & Harel, 1990). Hence, in the mathematics education literature, in spite of the procedural links between addition and multiplication, these two forms of reasoning are too distinct to be considered as a single conceptual domain (Nunes & Bryant, 2010 ; also see Van Dooren, De Bock, & Verschaffel, 2010; Vergnaud, 1988).

Research on additive and multiplicative reasoning has mainly developed separately (Greer, 1992; Van Dooren, De Bock, & Verschaffel, 2010). The present study brings together both lines of research, thereby focusing on additive and multiplicative missing-value word problems. In the following section, we elaborate on two types of errors that children often make: (1) solving multiplicative missing-value word problems additively and (2) additive missing-value word problems multiplicatively. In doing so, we describe and challenge the way in which the modelling perspective traditionally explained these errors, that is as being due to lacking skills, and we propose a complementary explanation in terms of a preference for additive or multiplicative relations.

2. Theoretical and empirical background

2.1 Solving missing-value word problems: the modelling perspective

Mathematical modelling is thought of as a complex process of applying appropriate mathematical operations to make sense of everyday life situations, consisting of several phases (Blum & Niss, 1991; English & Lesh, 2003; Van Dooren, Verschaffel, Greer, & De Bock, 2006; Verschaffel, Greer, & De Corte, 2000; Verschaffel et al., 2007). Children need to understand the problem situation, build the situation model, translate it into a mathematical model, conduct computations on this mathematical model, interpret and evaluate the outcome of these computations, and finally, report the result (Verschaffel et al., 2000). In this mathematical modelling process, two central skills – which seem particularly important in children’s errors on which we focus in the present study – can be distinguished, namely the skill to analyze the quantitative relations involved in the problem, which involves discriminating between different types of relations in order to determine the correct computation in the problem at hand (i.e., discrimination skill), and the skill to correctly execute those computations (i.e., computation skill). Within this modelling literature it has been shown that many children do not take such a genuine modelling approach. They often immediately jump from the problem statement to the identification of the underlying mathematical model merely by attending to superficial cues, and afterwards directly report the results of the computations conducted on this model without any further interpretation and verification (Van Dooren et al., 2006; Verschaffel et al., 2000, 2007).

Although mathematical modelling reaches far beyond simple school word problems, missing-value word problems are generally considered as suitable simple exercises in mathematical modelling (Verschaffel et al., 2000, 2007). Consequently, children’s additive errors in multiplicative missing-value word problems and multiplicative errors in additive missing-value word problems have been considered as a specific instance of the absence of a genuine modelling approach (Van Dooren et al., 2006). In the next section, we will present empirical research documenting both types of errors, and elaborate on their explanation in terms of (lacking) computation and discrimination skills.

2.2 The additive error in multiplicative missing-value word problems

Numerous studies showed that children frequently produce additive solutions to multiplicative missing-value problems (e.g., Hart, 1981; Kaput & West, 1994; Karplus et al., 1983; Lesh et al, 1988; Noelting, 1980; Vergnaud, 1983, 1988). For instance, children erroneously answer 12 miles instead of 24 to the car problem introduced before (2+4=6, so 8+4=12). This error is mainly made by younger children (e.g., in third or fourth grade in Flanders) and in missing-value problems wherein the ratios between the given numbers form non-integer ratios (Hart, 1981; Kaput & West, 1994; Karplus et al., 1983; Lesh et al. 1988; Tourniaire & Pulos, 1985; Van Dooren, De Bock, Evers, & Verschaffel, 2009; Vergnaud, 1983, 1988). Especially the nature of the internal ratio has been found to have the largest impact on children's solutions: Non-integer internal ratios, as compared to integer ones, evoke less correct multiplicative answers (Karplus et al., 1983; Van Dooren et al., 2009) and more additive errors (Karplus et al., 1983). Performance has been found to be even worse if both ratios are non-integer. This number effect has been found to decrease across grades (Karplus et al., 1983; Van Dooren et al., 2009).

The finding that especially younger children solve multiplicative problems additively has been interpreted as evidence for "an 'additive phase' in children's solution to multiplicative reasoning problems" (Nunes & Bryant, 2010, p. 11), in which children are not yet able to think of relations in a multiplicative way (Clark & Kamii, 1996; Siemon, Breed, & Virgona, 2005). Inhelder and Piaget (1958) already assumed that children can at first only quantify relations in an additive way – by focusing on the "equality of differences" (p. 177) – and only later, after reaching the formal-operational level of cognitive functioning, they can think of multiplicative relations too. This "transition from additive to multiplicative thinking" has been depicted as "one of the major barriers to learning mathematics in the middle years", or as "the big challenge of the middle years" (Siemon et al., 2005, p. 1). Hence, those young children still had to develop the skills of doing multiplicative computations, and of discriminating between multiplicative and additive relations in problem situations. As Lamon (2008) stated, "it takes some degree of mathematical maturity to understand the difference between adding and multiplying and contexts in which each operation is appropriate. […] Children who cannot yet tell the difference indiscriminately employ additive transformations." (p. 7).

However, this sequential development "first-additive-then-multiplicative-reasoning" has been questioned, based on evidence for children's multiplicative reasoning in the first years of primary school in a diversity of daily-life situations: sharing candies over persons, matching proportional mixtures of juice and water, comparing the size of pieces of multiple pies sliced in a different number of pieces, etc. (e.g., Boyer, Levine, & Huttenlocher, 2008; Jeong, Levine, & Huttenlocher, 2007; Nunes & Bryant, 2010). Those children are able to execute multiplicative operations to derive the correct answer, as well as to correctly discriminate between additive and multiplicative relations. Both skills – computation and discrimination skill – are crucial to multiplicative reasoning (Nunes & Bryant, 2010; Nunes, Bryant, Barros, & Sylva, 2012; Nunes, Bryant, Evans, & Barros, 2015).

2.3 The multiplicative error in additive missing-value word problems

Besides solving multiplicative problems additively, the inverse error has been observed as well. Children making this error answer "45 laps" to the aforementioned runner problem (i.e., 3×5=15, so 9×5=45). This error has been found mainly in upper primary education (Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2012; Van Dooren, De Bock, & Verschaffel, 2010), and in missing-value problems wherein the numbers form integer number ratios (Van Dooren et al., 2009; Van Dooren, De Bock, & Verschaffel, 2010). As for the inverse mistake of erroneously solving multiplicative word problems additively, the internal ratio has been found to have the largest impact here too. Integer internal ratios, as compared to non-integer ones, evoke less correct additive answers and more multiplicative errors. Performance has been found to decrease even further when neither of the ratios is an integer (Van Dooren et al., 2009). Moreover, between the age groups wherein children solve multiplicative word problems additively and additive word problems multiplicatively, many children have been found to pass through an intermediate stage, wherein they simultaneously make both types of errors, relying on the ratios between given numbers: A multiplicative solution is given when both ratios are integer; an additive solution when they are non-integer (Van Dooren, De Bock, & Verschaffel, 2010).

Analogously to solving multiplicative word problems additively, many upper primary school children who solve additive word problems multiplicatively are assumed to be able to reason additively.

Additive computation is intensively practiced from the start of primary education on, and the occurrence of additive errors at younger age demonstrates children's additive computation skill. While the literature documenting multiplicative errors in additive word problems does not seem to be primarily concerned about those children's skill to make additive computations, major concerns have been raised about children's discrimination skill. For instance, Hoffer (1988) argued that being able to perform multiplicative mechanical operations does not necessarily mean that the children understand the underlying ideas of multiplicative reasoning. Nevertheless, recent research evidence has demonstrated that upper primary school children perform better at classifying additive and multiplicative word problems than at simply solving them (Van Dooren, De Bock, Vleugels, & Verschaffel, 2010). This suggests that many children have acquired the skill of identifying an additive situation as being additive, and hence of discriminating between additive and multiplicative relations underlying word problems.

2.4 Explaining erroneous word problem solving behavior: beyond skills

While children's systematic errors in missing-value word problems have been considered as a specific instance of the absence of a genuine modelling approach in children, this does not seem to suffice to fully explain these errors. Given that the modelling perspective focuses on what is absent here, that is genuine modelling behavior, it does not provide a sufficient explanation for the systematic occurrence of *this* specific kind of errors. The modelling perspective may particularly be unable to fully account for the *individual differences* in this specific erroneous behavior. These individual differences occur within grade levels, implying that some children with a similar educational background (including curricula, educational approaches, and sociomathematical norms) make additive errors while others make multiplicative errors. Also the skills needed to do the required computations and to discriminate between additive and multiplicative situations seem not sufficient to guarantee genuine modelling behavior. Children may have acquired these two skills, which are central in the mathematical modelling perspective, but still give erroneous answers. Hence, when explaining children's erroneous word problem solving behavior, we need to look for an additional explanatory element, in addition to lacking computation and discrimination skills. Such an additional explanatory element was raised by Resnick and Singer (1993). They interpreted children's additive solutions to multiplicative missing-value word problems as an indication of their "preference for additive answers to proportion problems" (p. 123).

Likewise, children's multiplicative solutions to additive missing-value word problems may be explained by *a preference* for multiplicative relations. More recently, children's errors in missing-value problems have been explained as "tendencies" (Van Dooren, De Bock, & Verschaffel, 2010) or "inclinations" (Modestou & Gagatsis, 2010) towards additive or multiplicative relations. In the present study, we use the term "preference". This term expresses that children – either deliberately or not – are more strongly attracted towards one relation than to the other. Or, as Pellegrino and Glaser (1982) stated, that one type of relation "has precedence over" the other (p. 310).

2.5 Previous research on children's preference

Numerous studies in the broader domain of psychology of mathematics education made a similar distinction between preference and skill. This distinction rests on the idea that preference and skill do not fully coincide, although they may relate to and interact with each other (e.g., see Bailey, Littlefield, & Geary, 2012; Pellegrino & Glaser, 1982). Much research has been devoted to preferences for certain strategies in single-digit additions and subtractions (fact retrieval or counting; Bailey et al., 2012), in multi-digit additions and subtractions (mental computation or written arithmetic; Lemaire & Siegler, 1995; Torbeyns & Verschaffel, 2013), and in multiplication and division problems (repeated addition, counting, or derived fact; Mulligan, 1992). Even in more advanced mathematical content domains, high school students and mathematical experts demonstrate a preference for certain ways of processing calculus problems (visually or analytically; Haciomeroglu, Chicken, & Dixon, 2013), for certain types of arguments leading to proofs (empirically grounded, algebraic or geometrical; Greer, De Bock, & Van Dooren, 2009), or for certain solution approaches to word problems (algebraic or arithmetical; Van Dooren, Verschaffel, & Onghena, 2002).

In those studies, preference is typically measured by means of tasks that allow one to use a variety of strategies (Bailey et al., 2012; Mulligan, 1992; Torbeyns et al., 2013), ways of processing (Haciomeroglu et al., 2013), or solution approaches (Van Dooren et al., 2002). Evidence for children's preference was derived, when children (or adults) repeatedly used their preferred strategy or solution approach, despite having the choice to use others. An alternative approach to measure preference, which so far has been used less often in the broader domain of the psychology of mathematics education, is asking to score or rank several solution approaches (Van Dooren et al., 2002) or proofs (Greer et al., 2009; also see Inglis & Aberdein, 2015). The latter approach, of course, supposes that preference rests on a deliberate choice. However, as outlined above (in section 2.4), for the preference that is the focus of the current study this is not necessarily the case. Hence we opt for the first, and most commonly used, approach to measure preference.

Although we argued that children's preference is at play in their (erroneous) word problem solving behavior, "classical" word problems may not be best suited to capture children's preference. They unmistakably contain an underlying additive or multiplicative mathematical model, and therefore do not allow children to use a variety of (correct) ways of reasoning. Hence, children's skills to analyze the relations underlying the problem and discriminate between additive and multiplicative ones may be involved in solving those word problems too apart from their preference. Measuring children's preference requires problems without any indication for an additive or multiplicative solution.

Several authors have worked with problems that are *open* to different solution methods, interpretations, or answers (e.g., Schukajlow, Krug, & Rakoczy, 2015; Silver, 1995). In the domain of additive and multiplicative reasoning in particular, several problems in which both additive and multiplicative reasoning are equally valuable and correct, have been used (e.g., Lamon & Lesh, 1992, Nunes & Bryant, 2010; Pellegrino & Glaser, 1982). As Lamon (2008) stated it: "the message here is *not* that one perspective is wrong and the other is correct. Both perspectives are useful" (p. 32). One example of such an open problem that has been shown to validly measure children's preference for additive or multiplicative relations between numbers, is a schematic problem consisting of three given numbers and a fourth missing one, and two arrows that point out the relational structure between numbers (see Degrande, Verschaffel, & Van Dooren, 2018). This schematic problem is considered open, since those number pairs "can be represented as having several relationships" (e.g., the pair 2 and 16 can be represented as having the following relationships: +14, ×8 or even others such as 2^4, see Pellegrino & Glaser, 1982, p. 302). Hence, answers based on additive or multiplicative relations (and also other answers) are equally correct and valuable.

Previous studies using such open problems revealed that many third to sixth graders preferred additive relations, as originally suggested by Resnick and Singer (1993), whereas others preferred multiplicative relations. This variety in preferences was not only found when children were presented with open problems in which they had to fill out an additive or multiplicative answer, but even in similar problems in which they were asked to indicate all possibly correct answers among a set of given alternatives including the additive and multiplicative answers. While inter-individual differences in the preferred relations were found within each grade, mainly younger children preferred additive relations while most older children preferred multiplicative relations. Children preferred additive relations mainly in problems in which both ratios were non-integer and multiplicative ones in problems in which both ratios were integer. In addition to that, it has been shown that preference is related to, but does not coincide with, computation skill (Degrande, Verschaffel, & Van Dooren, 2018).

2.6 The present study

While the aforementioned findings using open problems strongly resembled findings of “classical” word problem research – suggesting that children’s preference may be at play in word problem solving – the impact of preference on children’s erroneous word problem solving behavior has not been explicitly tested yet. Therefore, the present study investigated to what extent children’s errors in additive and multiplicative word problems can be uniquely explained by means of a preference for additive or multiplicative relations, besides lacking additive and multiplicative reasoning skills. We measured word problem solving behavior, preference, and computation and discrimination skills.

By administering those four instruments, we aimed to *replicate and systematize* previous research results: To what extent do children show erroneous additive or multiplicative word problem solving behavior in missing-value word problems, and how is this affected by grade (RQ1), and to what extent do children prefer either additive or multiplicative relations, and how is this affected by grade (RQ2)? More importantly, we also aimed to *extend* those results by explaining this erroneous word problem solving behavior: To what extent can children’s preference for additive or multiplicative relations explain their erroneous word problem solving behavior, besides skills (RQ3)? Both research questions are separately answered for items containing integer or non-integer number ratios. This problem characteristic strongly affects children’s word problems solving behavior, since it is harder to calculate the correct (multiplicative) solution in problems containing non-integer than integer ratios (e.g., see Van Dooren et al., 2009). Therefore, a smaller group of children may have acquired the computation skill in non-integer as compared to integer problems.

3. Method

3.1 Participants

Participants were 246 children in third to sixth grade from three primary schools in Flanders (Belgium). The sample consisted of 68 third (*M* = 104.82 months, *SD* = 3.82 months), 59 fourth (*M* = 119.02 months, *SD* = 4.71 months), 58 fifth (*M* = 128.71 months, *SD* = 3.94 months), and 61 sixth graders (*M* = 142.18 months, *SD* = 5.39 months). About half of the children were boys (*n* = 118). The sample scored above average (as compared to a national sample) on a general math achievement test (*M* = percentile 64.1, *SD* = 27.6 percentiles), as measured by a curriculum-based standardized test from the Flemish student monitoring system (Deloof, 2005; Dudal, 2002, 2003; Dudal & Deloof, 2004). All three schools were situated in villages, were average in size (ranging from about 200 to about 450 children per school), and attracted children from diverse socio-economic backgrounds (with the percentages of children from low socio-economic backgrounds ranging from about 3.6% to about 24.2%, based on the Flemish education allowance).

3.2 Instruments

Four test instruments were developed: a word problem test, a preference test, a computation skill test and discrimination skill test. Several versions of those test instruments, containing different but comparable problem situations (all within the context of speed and distance) and number combinations (all leading to an integer outcome that was smaller than 100) in different orders, were used to minimize the impact of specific item characteristics. Regarding the number combinations, both the internal and external number ratios were either integer or non-integer, given that previous research on additive or multiplicative word problem solving has shown that the simultaneous manipulation of both ratios has the largest impact on children’s answers (e.g., Karplus et al., 1983; Van Dooren et al., 2009). All integer number combinations contained ratios of 2, 3, or 4, and non-integer number combinations contained ratios of 1.5, 2.5, or 3.5. In what follows, the four test instruments are presented.

A first test instrument, the word problem test (WPT) contained 12 missing-value word problems that were extensively used in previous research by several scholars (e.g., Fernández et al., 2012; Van Dooren, De Bock, Vleugels, et al., 2010). Children were asked to write down their calculations and final solution. Six word problems were additive (see Figure 1a), the other six were multiplicative (see Figure

1b). The additive and multiplicative word problems contained comparable situations, all within the context of speed and distance. Amongst both problem types, various specific formulations were counterbalanced (e.g., the use of the words “earlier” or “later” for additive problems and “faster” or “slower” in multiplicative problems), to avoid that children would choose their solution based on superficial context characteristics or problem formulations. Amongst additive and multiplicative problems, three problems containing integer number ratios and three problems containing non-integer number ratios were presented to children. Integer and non-integer word problems contained similar number combinations with the same first value, and comparable other values within the same size range.

Figure 1a. **Example of integer additive word problem**	**Figure 1b.** **Example of integer multiplicative word problem**
Ellen and Kim are running around a track. They are running equally fast, but Ellen started later. When Ellen has run 4 laps, Kim has run 16 laps. When Ellen has run 8 laps, how many laps has Kim run? Answer: Kim has run … laps. Calculation:	Lynn and Julie are reading the same book. They started at the same time, but Julie reads faster. When Lynn has read 4 pages, Julie has read 16 pages. When Lynn has read 8 pages, how many pages has Julie read? Answer: Julie has read … pages. Calculation:

Figure 1. Examples of test items of the WPT.

A second test instrument was the preference test, which consisted of 12 schematic problems containing three given numbers and a fourth missing one, as well as two arrows pointing out the relational structure. Since the additive and multiplicative answer were equally correct and valuable, these problems were considered *open* problems. They have been shown to validly measure children’s preference in previous research (Degrande, Verschaffel, & Van Dooren, 2018). Half of the open problems contained integer number ratios (see Figure 2a), the other half contained comparable non-integer number ratios (see Figure 2b). This number of items allowed us to measure children’s preference reliably, without stimulating response tendencies across problems within this test instrument.

Figure 2a.
Example of integer open problem

Look at the first arrow.
Do the same for the second arrow now.
Which number comes in the empty box?

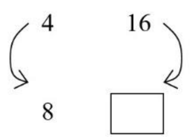


Figure 2b.
Example of non-integer open problem

Look at the first arrow.
Do the same for the second arrow now.
Which number comes in the empty box?

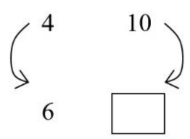


Figure 2. Examples of test items of the preference test.

A third test measured the additive and multiplicative computation skill. This test consisted of the same 12 schematic problems of the preference test, but this time accompanied by addition or multiplication signs indicating the calculation children had to execute. Six additive (see Figure 3a) and six multiplicative items (see Figure 3b) were offered, and each of the two types consisted of three items containing integer number ratios and three items containing non-integer number ratios.

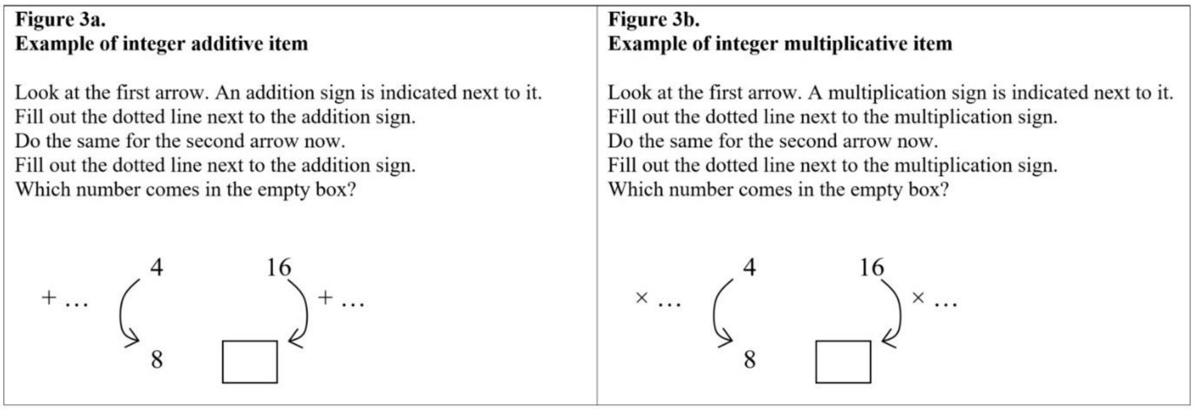


Figure 3. Examples of test items of the computation skill test.

A fourth test, measuring the additive and multiplicative discrimination skill, contained the same 12 word problems as the WPT (six additive and six multiplicative ones, see Figure 4a and 4b respectively). Children were asked which of both correctly completed solution schemes (i.e., additive or multiplicative) fitted the given word problem.

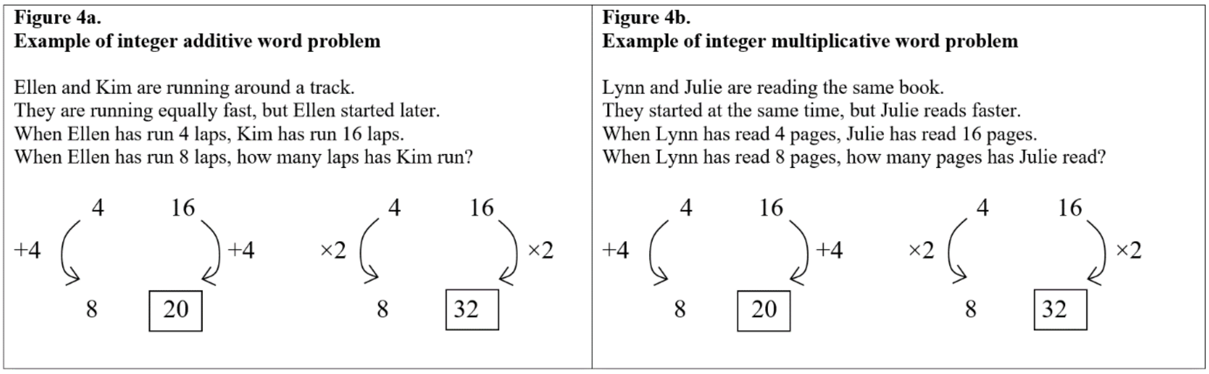


Figure 4. Examples of test items of the discrimination skill test.

Furthermore, general mathematics achievement was measured by means of the Flemish math achievement test which is typically used by schools to monitor children's progress throughout primary school. This test contains 60 items covering several aspects of the mathematics curriculum. Percentile scores per grade were included as measure of general math achievement.

3.3 Procedure

The four tests were administered on two separate moments with max. one week in between both, to ensure optimal concentration of participants. The preference test and WPT were administered in this order at the first testing moment, and the computation and discrimination skill tests in this order at the second moment. This order was chosen to avoid carry-over effects from the skill tests to the WPT, and especially to the preference test. No time limit was given. The instructor told that the tests aimed at assessing general mathematics achievement. To get children acquainted with the problem format and the instructions, both tests measuring additive and multiplicative reasoning skills (i.e., computation and discrimination skill) were preceded by some collectively solved practice trials. The correct answer was not explicitly discussed, to avoid an impact of the practice trials on children's actual additive or multiplicative reasoning skill.

3.4 Analyses

Answers of all test instruments were coded as additive, multiplicative, or other, if children solved the tasks additively, multiplicatively, or using still another solution (including leaving the problem unanswered). The code "additive-and-multiplicative" was given if children solved the tasks using a combination of both an additive and a multiplicative answer to the same item. This code was especially expected to occur in the preference test, where both answers were equally correct and valuable. Answers containing calculation errors were scored as additive or multiplicative, provided that written calculations could unequivocally be characterised as additive or multiplicative. This was not the case in the computation skill test (where obtaining the correct numerical answer via calculations was exactly the object of study), nor in the discrimination skill test (where no calculations were required to select the correct solution scheme that fitted the problem).

To answer RQ1 and RQ2, frequency histograms of children's erroneous answers to the WPT, and of children's answers to the preference test were constructed. Kruskal-Wallis tests were further conducted to test the impact of grade on those variables. To answer RQ3, first, the extent to which children had acquired the additive and multiplicative computation and discrimination skills was examined. Performances on the various skill tests were compared by means of Wilcoxon signed rank tests. Further, children who answered *all* additive or multiplicative computation items correctly, and children who answered *all* additive or multiplicative discrimination items correctly, were selected. This strict criterion yields the most convincing evidence that the selected children had acquired the additive or multiplicative computation skills, and additive or multiplicative discrimination skills, respectively. Second, the relation between additive preference and the additive error in multiplicative word problems was investigated for all children who had acquired the multiplicative computation and discrimination skills, while the relation between multiplicative preference and the multiplicative error in additive word problems was investigated for all children who had acquired the additive computation and discrimination skills. This was done by using hierarchical regression analyses, in which we included as a first step several control variables that have been found to be associated with additive and multiplicative reasoning in word problems: age (e.g., Tourniaire & Pulos, 1985), math achievement (e.g., Nunes et al., 2012), and gender (e.g., Tourniaire & Pulos, 1985). Preference was added in a second step, because we were interested in the unique additional contribution of preference to erroneous word problem solving behavior.

4. Results

In a first part, we look at results on the *integer* items of all tests. In a second part, we do the same for the *non-integer* items of all tests. This problem characteristic strongly affects children's word problems solving behavior, since it is harder to calculate the correct multiplicative solution in non-integer than integer items. Hence, a smaller group of children may have acquired the computation skill in non-integer, as compared to integer items.

4.1 Integer items

Word problem test. Figure 5 presents a frequency histogram of the erroneous answers to the WPT, showing that 154 children (62.6%) answered at least one additive word problem multiplicatively, and 84 children (34.1%) answered *all* additive word problems multiplicatively. Likewise, 98 children (39.8%) answered at least one multiplicative word problems additively, while 46 children (18.7%) did so for *all* multiplicative word problems. This indicates that these problems evoked many errors of both types, and also that these errors occurred with some degree of consistency within children.

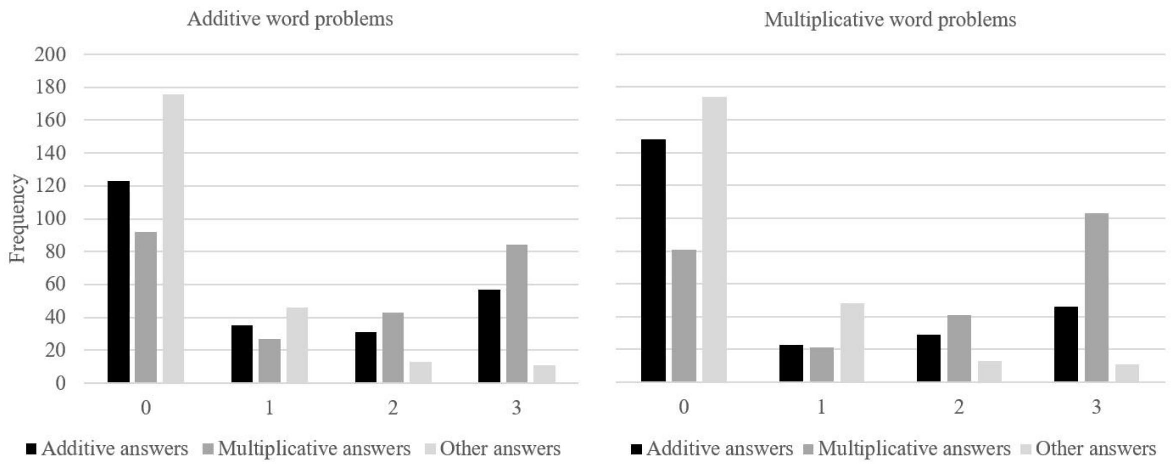


Figure 5. Frequency histogram of additive, multiplicative, and other answers to the integer WPT.

Second, while we found individual differences in erroneous word problem solving behavior within each grade, the occurrence of both errors differed across grades in line with previous research. Fifty third graders (73.5%) answered at least one, and 24 (35.3%) answered all multiplicative word problems additively, but this decreased to 7 (11.5%) and 2 (3.3%) sixth graders, respectively. Likewise, 21 children (30.9%) answered at least one and 4 (5.9%) answered all additive word problems multiplicatively in third grade, but this increased to 50 (82.0%) and 34 (55.7%) children, respectively, in sixth grade. Kruskal-Wallis tests confirmed the decrease of the additive error problems ($\chi^2(3)=58.068$, $p<.001$), and the increase of the multiplicative error across grades ($\chi^2(3)=57.208$, $p<.001$).

Preference test. First, 109 children (44.3%) gave at least one additive answer to the preference test, and 17 children (6.9%) answered all open problems additively (see Figure 6). Likewise, 184 children (74.8%) answered at least one open problem multiplicatively, and 79 children (32.1%) did so for all problems in the preference test. We could not identify any “additive-and-multiplicative” answers. This not only indicates that the preference test evoked both additive and multiplicative answers, but also that these answers were given quite consistently within children, and that there were no occasions where children raised both answers for the same problem. Hence, to answer RQ2, our results revealed that children indeed tended to prefer additive or multiplicative relations.

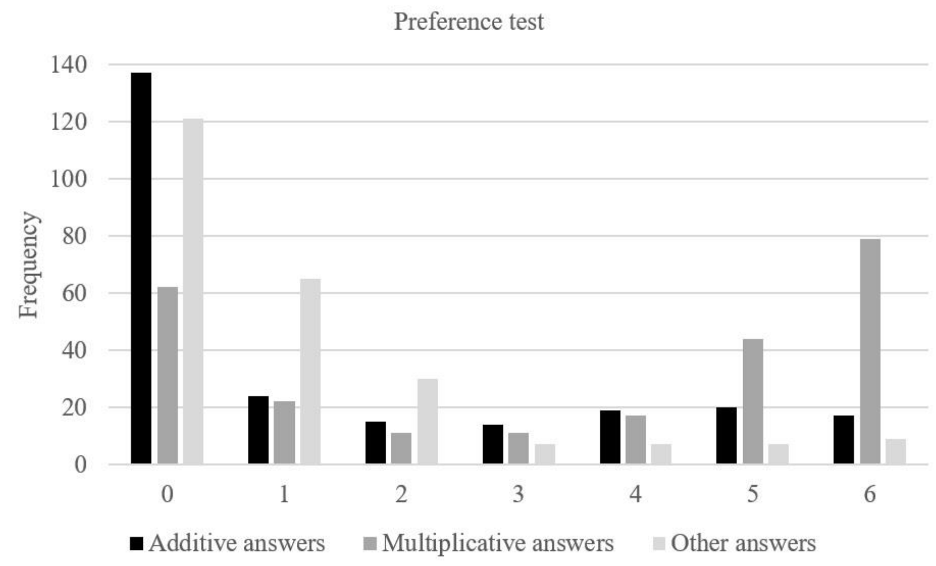


Figure 6. Frequency histogram of additive, multiplicative, and other answers to the integer preference test.

Second, while we found individual differences in additive and multiplicative preference within each grade, the occurrence of both preferences differed between grades. In third grade, 51 children (75.0%) answered at least one, and 8 (11.8%) answered all open problems additively, but this decreased to 11 (18.0%) and 3 (4.9%) children, respectively, in sixth grade. Likewise, 30 third grades (44.1%) answered at least one, and 1 third grader (1.4%) answered all open problems multiplicatively, but this increased to 57 (93.4%) and 35 (57.4%) sixth graders, respectively. Kruskal-Wallis tests confirmed the decrease of the additive preference ($\chi^2(3)$= 47.081, p<.001), and the increase of the multiplicative preference across grades ($\chi^2(3)$=89.636, p<.001).

Selection of children based on computation and discrimination skill test. To answer RQ3, we selected the children who had acquired the necessary computation skills. For children who answered multiplicative word problems additively, it was especially relevant to check whether they had acquired the multiplicative computation and discrimination skills. Overall, multiplicative computation skills were much better acquired than multiplicative discrimination skills (see Figure 7, Z=-7.902, p<.001). This led to a selection of 165 children who had acquired the multiplicative computation skill (67.1%), 91 who had acquired the multiplicative discrimination skill (37.0%), and 77 who possessed both multiplicative reasoning skills (31.3%). This latter group was included in further analyses of erroneous additive reasoning in multiplicative problems.

Additive computation skills were also better acquired than additive discrimination skills (*Z*=-9.322, *p*<.001). This resulted in a selection of 201 children who had acquired the additive computation skill (81.7%), 88 who had acquired the additive discrimination skill (35.8%), and 75 who possessed both additive skills (30.5%). The latter group was included in further analyses of erroneous multiplicative reasoning in additive word problems. When comparing additive and multiplicative skills, it stood out that additive computation skills were overall better acquired than multiplicative computation skills (*Z*=-4.291, *p*<.001), while we could not find a difference between additive and multiplicative discrimination skills (*Z*=-1.028, *p*=.306).

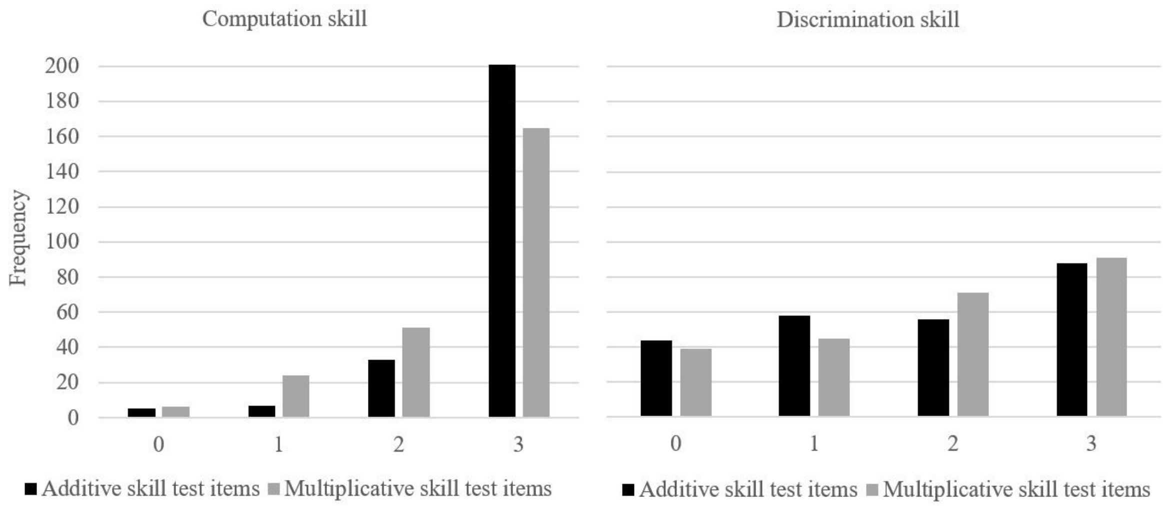


Figure 7. Frequency histogram of correct answers to integer additive and multiplicative computation and discrimination skill tests.

Preference in relation to word problem test. Within the two groups that had acquired the necessary skills, we conducted hierarchical regression analyses to investigate the relation between preference and erroneous word problem solving behavior (see Table 1). Age, general math achievement, and gender were added as a first step, and preference as a second step. In both types of erroneous word problem solving, preference was a unique and strong predictor. Preference for additive relations accounted for 27.4% of the explained variance (i.e., about two thirds) in erroneous additive word problem solving behavior ($F(4,71)=13.285$, $p<.001$, Total R^2=42.8%), while preference for multiplicative relations accounted for 11.6% (i.e., about half) of the explained variance in erroneous multiplicative word problem solving behavior ($F(4,68)=5.762$, $p<.001$, Total R^2 = 25.3%).

Table 1

Regression models predicting the additive error in multiplicative word problems (1a) and the multiplicative error in additive word problems (1b)

Table 1a

Step	Predictor	Mean	*SD*	Range	*n*	Standardized *β*	R^2 change
1	Age	129.58	11.7	103-160	77	-.155	.154
	Math achievement	75.51	24.2	15-99	76	-.087	
	Gender (Male)	0.57	0.50	0-1	77	-.083	
2	Additive preference	0.64	1.50	0-6	77	.561***	.274

Note Table 1a. $n = 76$, *$p < .05$, **$p < .01$, *** $p < .001$

Table 1b

Step	Predictor	Mean	*SD*	Range	*n*	Standardized *β*	R^2 change
1	Age	121.75	13.55	96-145	75	-.108	.137
	Math achievement	76.93	23.66	1-99	73	.140	
	Gender (Male)	0.63	0.49	0-1	75	.140	
2	Multiplicative preference	3.48	2.55	0-6	75	.436**	.116

Note Table 1b. $n = 73$, *$p<.05$, **$p<.01$, ***$p<.001$

4.2 Non-integer items

Word problem test. Figure 8 presents a frequency histogram of the erroneous answers to the word problem test, showing that 70 children (28.5%) answered at least one additive word problem multiplicatively, and 35 children (14.2%) did so for all additive word problems. Likewise, 170 children (69.1%) answered at least one multiplicative word problem additively, and 116 children (47.2%) did so for all multiplicative word problems. This indicates that these problems not only evoked many errors of both types, but also that these incorrect answers occurred consistently within children.

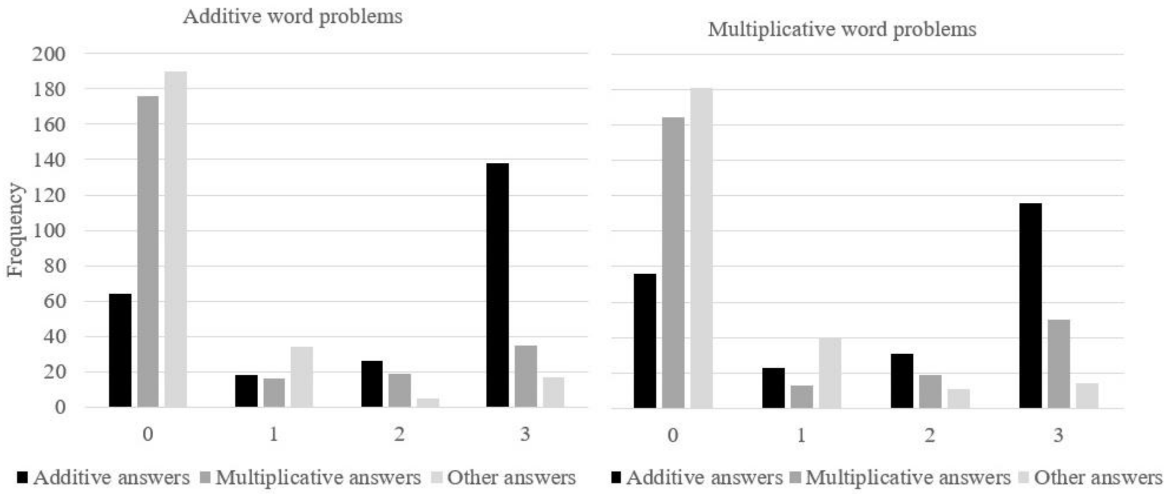


Figure 8. Frequency histogram of additive, multiplicative, and other answers to the non-integer WPT.

Second, while we found individual differences in erroneous word problem solving behavior within each grade, the occurrence of both errors differed across grades in line with previous research. In third grade, 57 children (83.8%) answered at least one, and 41 (60.3%) answered all multiplicative word problems additively, but this decreased to 24 (39.3%) and 15 (24.6%) children, respectively, in sixth grade. Likewise, while 3 (4.4%) third graders answered at least one and no children at all answered all additive word problems multiplicatively, this increased to 39 (63.9%) and 21 (34.4%) sixth graders, respectively. Kruskal-Wallis tests confirmed the decrease of the additive error ($\chi^2(3)=38.075$, $p<.001$), and the increase of the multiplicative error across grades ($\chi^2(3)=65.440$, $p<.001$).

Preference test. First, 193 children (78.5%) gave at least one additive answer to the preference test, and 97 children (39.4%) answered all open problems additively (see Figure 9). Sixty-nine children (28.0%) answered at least one open problem multiplicatively, and 24 children (9.7%) did so for all problems in the preference test. We could not identify any "additive-and-multiplicative" answers.

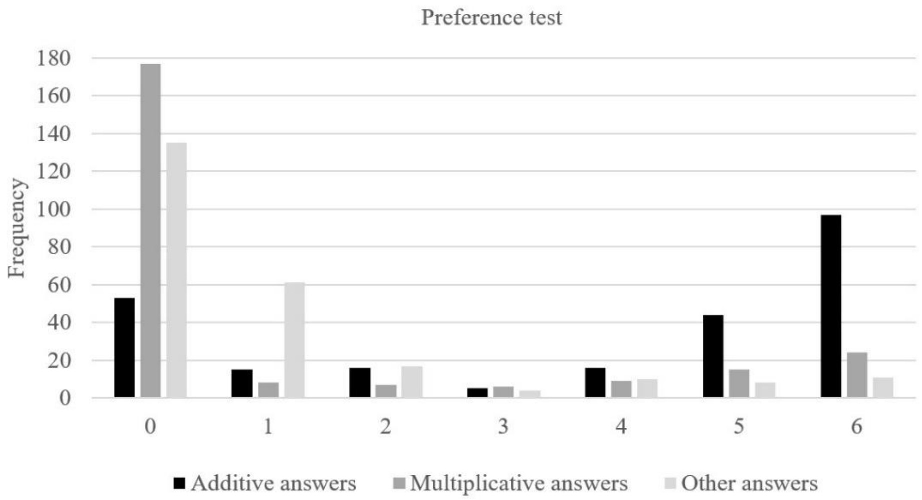
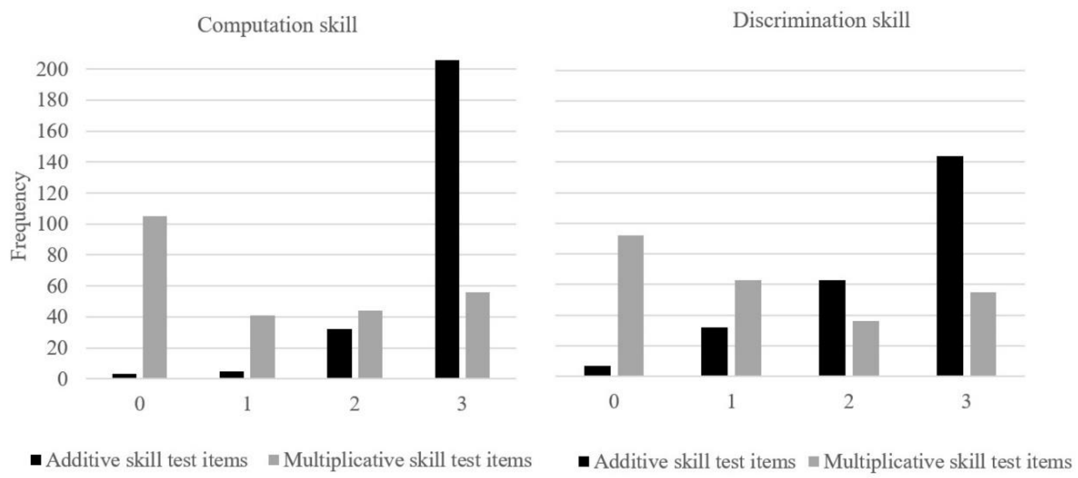


Figure 9. Frequency histogram of additive, multiplicative, and other answers to the non-integer preference test.

Second, while we found individual differences in additive preference and in multiplicative preference within each grade, the occurrence of both preferences differed between grades. Fifty-eight third graders (85.3%) answered at least one and 29 (42.6%) answered all open problems additively, but this decreased to 34 (55.7%) and 16 (26.2%) sixth graders. Likewise, 3 children (4.4%) answered at least one and no children answered all open problems multiplicatively in third grade, but this increased to 38 (62.3%) and 14 (23.0%) children in sixth grade. Kruskal-Wallis tests confirmed the decrease of the additive ($\chi^2(3)=16.910$, $p=.001$) and the increase of the multiplicative preference ($\chi^2(3)=66.714$, $p<.001$).

Selection of children based on computation and discrimination skill test. To answer RQ3, we selected those children who had acquired the necessary computation skills. For children who answered multiplicative word problems additively, it was especially relevant to check whether they had acquired the multiplicative computation and discrimination skills. A Wilcoxon signed rank test indicated that performance on both multiplicative skills tests was equally low (see Figure 10, $Z=-.134$, $p=.893$). This led to a selection of 56 children who had acquired the multiplicative computation skill (22.8%), 55 who had acquired the multiplicative discrimination skill (22.4%), and 23 who possessed both multiplicative skills (9.3%). This latter group was included in further analyses of erroneous additive reasoning in multiplicative word problems. Additive computation skills were in general better acquired than additive discrimination skills ($Z=-6.234$, $p<.001$). This resulted in a selection of 206 children who had acquired the additive computation skill (83.7%), 144 who had acquired the additive discrimination skill (58.5%), and 122 who possessed both additive skills (49.6%). The latter group was included in further analyses of erroneous multiplicative reasoning in additive word problems. When comparing additive and multiplicative skills[1], it stood out that additive computation skills were better acquired than multiplicative computation skills ($p<.001$), and that additive discrimination skills were better acquired than multiplicative discrimination skills ($p<.001$) in those non-integer items.



[1] A sign test was used here, due to an asymmetrical distribution of differences between the two related scores.

Figure 10. Frequency histogram of correct answers to the non-integer additive and multiplicative computation and discrimination skill tests.

Preference in relation to word problem test. Within the groups that had acquired the necessary skills, we conducted hierarchical regression analyses to investigate the relation between preference and erroneous word problem solving behavior (see Table 2). Age, general math achievement, and gender were added as a first step, and preference as a second. In both types of erroneous word problem solving, preference was a unique predictor. Preference for additive relations accounted for 52.7% (i.e. more than two thirds) of the explained variance in erroneous additive word problem solving behavior ($F(4,17)=12.589$, $p<.001$, $R^2=74.8$), and preference for multiplicative relations accounted for 19.6% (i.e. more than half) of the explained variance in erroneous multiplicative word problem solving behavior ($F(4,113)=16.200$, $p<.001$, $R^2=36.4$).

Table 2

Regression models predicting the additive error in multiplicative word problems (2a), and the multiplicative error in additive word problems (2b).

Table 2a

Step	Predictor	Mean	*SD*	Range	*n*	Standardized *β*	R^2 change
1	Age	131.26	10.60	110-145	23	-.125	.220
	Math achievement	88.91	12.53	45-99	22	-.339*	
	Gender (Male)	0.70	0.47	0-1	23	-.258	
2	Additive preference	1.74	2.34	0-6	23	.737***	.527

Note Table 2a. $n = 22$, *$p < .05$, **$p < .01$, *** $p < .001$

Table 2b

Step	Predictor	Mean	*SD*	Range	*n*	Standardized *β*	R^2 change
1	Age	125.21	13.79	100-150	122	.188*	.169
	Math achievement	71.58	24.58	5-99	118	.061	
	Gender (Male)	0.55	0.50	0-1	122	-.100	
2	Multiplicative preference	1.39	2.23	0-6	122	.502***	.196

Note Table 2b. $n = 122$, *$p < .05$, **$p < .01$, ***$p < .001$

5. Conclusion and discussion

5.1 Theoretical implications

The present study *replicated* previous results regarding erroneous word problem solving behavior, and *systematized* those results by combining the domains of additive and multiplicative reasoning. In line with previous studies, multiplicative word problems evoked many additive errors (e.g., Hart, 1981; Kaput & West, 1994; Karplus et al., 1983; Lesh et al, 1988; Noelting, 1980; Vergnaud, 1983, 1988), and additive word problems evoked numerous multiplicative errors (Fernández et al., 2012; Van Dooren, De Bock, & Verschaffel, 2010), and both errors occurred quite consistently within children (see RQ1). Moreover, as expected, the additive error in multiplicative word problems occurred most often in the lower grades and in items containing non-integer ratios (e.g., Hart, 1981; Kaput & West, 1994; Karplus et al., 1983; Lesh et al. 1988; Tourniaire & Pulos, 1985; Van Dooren et al., 2009; Vergnaud, 1983, 1988), whereas the multiplicative error in additive word problems occurred most often in upper primary education and in items containing integer ratios (Fernández et al., 2012; Van Dooren et al., 2009). Regarding RQ2, we found that some children preferred additive relations and others multiplicative ones, and that these types of relations were consistently preferred within children. Further, while we found individual differences in additive and multiplicative preference within each grade and each type of number ratios, additive preference mainly occurred in younger children and in non-integer problems, whereas multiplicative preference mainly occurred in older children and in integer problems. All of this replicated previous results about children's preference for additive or multiplicative relations in primary education (Degrande, Verschaffel, & Van Dooren, 2018).

This study also *extended* previous research by investigating to what extent the above-mentioned additive or multiplicative errors depend on children's *preference* for additive or multiplicative relations, as a complementary explanation to their lacking *skills* (RQ3). First, we selected children who had acquired the multiplicative computation and discrimination skills, and children who had acquired the additive computation and discrimination skills. This selection revealed that computation skills were overall better acquired than discrimination skills (except in the non-integer items were children performed equally low on both multiplicative skills), which confirmed at least partly scholars' claims regarding children's lacking additive and multiplicative discrimination skills (Lamon, 2008, Hoffer, 1988), or lacking multiplicative computation skills – specifically in non-integer items (e.g., Siemon et al., 2005). Moreover, the additive skills were overall better acquired than the multiplicative ones (except for the discrimination skill in integer items where we did not find any differences between additive and multiplicative skills). This latter finding not only confirms the commonly held assumption that additive reasoning is easier than multiplicative reasoning (e.g., Clark & Kamii, 1996), it also indicates that the additive error in multiplicative problems – which is most prominent in non-integer items – may to a larger extent be due to lacking multiplicative skills than that multiplicative error in additive problems – which is most prominent in integer items – would be due to lacking additive skills.

Second, for children who clearly had acquired both skills, we looked at *preference* in relation to erroneous word problem solving behavior. Both in problems with integer and non-integer ratios, our results revealed that additive preference uniquely explained erroneous additive word problem solving behavior, and that multiplicative preference uniquely explained erroneous multiplicative word problem solving behavior, after taking into account several relevant subject characteristics. This result stood in sharp contrast with the skill-based account of many previous studies, which mainly took a mathematical modelling perspective.

The present study was not specifically designed to identify educational factors that affect the origin of children's preference, since we have shown that preference explains incorrect word problem solving behavior in children who possess the necessary computation and discrimination skills. However, we suspect that an additive or multiplicative preference is the result of an implicit learning process in the context of additive and multiplicative relations. Additive missing-value word problems are highly absent in primary school curricula, while the prototypical missing-value word problems in upper primary education are multiplicative in nature and involve integer number ratios only. Also word problems that are open to both additive and multiplicative relations seem to be missing. Further, teachers' explanations and feedback are strongly focused on the fluent execution of computation skills, paying little or no attention to children's preference. The development of a preference may be further reinforced by implicit sociomathematical norms, such as the norms that every word problem has one and only one correct answer, and that the required mathematical operation is most likely the most recently taught one. Still, despite similar educational experiences, even within one grade children individually differed in preference. These differences in preference remained after selecting children who unmistakably had acquired the necessary computation and discrimination skills, and after taking into account age as measure for educational experience, and other subject characteristics related to preference (i.e., math achievement and gender). While preference seems related to other subject characteristics (e.g.

computation skill, see Degrande, Verschaffel, & Van Dooren, 2018), it does not coincide with these characteristics, and provides a unique explanation for the errors children commit in word problems.

5.2 Methodological implications

From a methodological perspective, the present study provided evidence for the validity of the preference construct in several ways. First, we still found individual differences in preference after selecting children with high levels of computation and discrimination skill. Preference has thus been identified as a construct that is distinct from both skills. This indicates the *discriminant validity,* because our measures allow to discriminate between constructs that are theoretically different (Cohen, Manion, & Morisson, 2018). Second, and even more importantly, amongst children who had acquired the necessary computation and discrimination skills, preference has been shown to account for systematic errors in word problems. The current preference test does exhibit some degree of *concurrent validity*, since answers on the preference and word problem tests are highly related (Cohen et al., 2018).

Besides this important methodological merit of our study, we identified several methodological limitations. First, a direct consequence of the subsequent selection of children based on the strictest criterion related to their acquired skills inevitably resulted in a limited number of children in the last step of our analysis. Even though our original sample size was substantial, this warrants some caution in generalizing those results. Another drawback of our analytic strategy is that we may have found more variation in erroneous word problem solving behavior and preference when considering the whole sample, as compared to the highly selective group of children who possessed the required skills.

Second, we focused on missing-value word problems dealing with one particular context (speed and distance). This poses restrictions to the generalizability of our findings to other problems within the domain of additive and multiplicative reasoning. One could wonder whether children’s errors – particularly erroneously solving multiplicative word problems additively – would occur as often, and especially whether preference would be as important in explaining this error, in missing-value word problems using other problems contexts such as price or mixtures, or multiplicative word problems with another structure (such as comparison problems in which four values are given and a judgment on proportionality is required). Likewise, future studies could find out whether preference, as measured by other open problem types, would still explain children’s missing-value word problem solving behavior, on top of their skills. These could, for instance, involve problems that are not only open to relational answers but also to answers referring to non-mathematical aspects of the situation (Degrande, Verschaffel, & Van Dooren, 2017), or open problems that have another format, such as open word problems (e.g., see Degrande, Van Hoof, Verschaffel, & Van Dooren, 2018; Lamon, 2008).

A third methodological consideration relates to the discrimination skill test. While it is not completely inevitable that preference may be partly at play in the discrimination test too, the mere presentation of the solution scheme containing numbers in the discrimination test may have triggered children's preference for additive or multiplicative relations between numbers, even though children did not have to conduct any calculations themselves. Future studies may use letters or symbols, or non-symbolic representations of numbers (e.g., a discrete group of dots, or a continuous line) instead. Alternatively, based on previous research on qualitative versus quantified multiplicative word problems (e.g. Heller, Ahlgren, Post, Behr & Lesh, 1989; Van Dooren, Vamvakoussi, & Verschaffel, 2018), expressing the relation in more qualitative terms without asking children to quantify the relations, such as in the runner problem "Compared to the first moment, Kim will have run (a) … *times as many* laps, (b)… *laps more* or (c) there is not enough information to tell" may stimulate them to focus on the quantities involved in the problem and the relations between them.

5.3 Educational implications

Notwithstanding the above-mentioned limitations and questions for further research, we can already cautiously derive some educational implications. Given that a preference for additive or multiplicative relations uniquely explains the occurrence of additive or multiplicative errors, it is worth considering how such a preference can be remedied and prevented. As will become clear below, such interventions may simultaneously also help in developing computation and especially discrimination skills. First, children should be brought to understand that many comparison and change situations can be viewed additively *as well as* multiplicatively (Lamon, 2008; Nunes & Bryant, 2010; Van Dooren et al., 2018). Problems open to both additive and multiplicative reasoning lend themselves for this purpose. Take, for instance, the following problem: "Suppose there are 12 girls and 18 boys in a class and they are assigned to single-sex groups during French lessons. If there were not enough books for all of them and the Head Teacher decided to give 4 books to the girls and 6 books to the boys, would this be fair?"

(Nunes & Bryant, 2010, p. 4). While children who prefer additive relations would answer that this is unfair (only eight girls but twelve boys girls don't have a book), children who prefer multiplicative relations would claim that this is fair (three boys will share one book and three girls will share one book). Second, educational activities should be targeted at realizing a conflict between children with different preferences in open problems, or between a child's preference and the mathematical model underlying word problems. Especially the latter may lead to the development of discrimination skills, and therefore initiate a process in which children may overcome the overreliance on their preferred type of relations. Classical word problems as well as open word problems can be used, and activities could focus on classifying word problems rather than actually solving them (e.g. Van Dooren, De Bock, Vleugels, et al., 2010), on problem posing activities (e.g., English, 1997; Lamon, 2008), or group discussion activities in which children are prompted to find, discuss and compare multiple solutions for a certain word problem (e.g., Lamon, 2008; Schukajlow et al., 2015). Third, these interventions should ideally be incorporated within a classroom culture in which sociomathematical norms that shape children's preference are made explicit and questioned, so that new ones may be gradually established. Taken altogether, it is not self-evident that teachers are aware of the existence of preference and its contribution to children's errors. They may even share children's preferences, and this may especially be the case for the multiplicative preference, considering the large number of multiplicative errors observed in preservice teachers too (Cramer et al., 1993). Hence, the awareness of the existence of preferences needs to be an integral part of teachers' pedagogical content knowledge.

Such activities addressing children's preference in the broader context of solving problems involving additive and multiplicative relations will also further stimulate the development of their computation and discrimination skills. Children will practice to execute additive and multiplicative calculations (also involving non-integer ratios) correctly and fluently, and they will meet a greater variety of word problems (incl. in terms of contexts, structures, formats, and types of ratios), which may help in developing better discrimination skills.

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Highlights

- Upper primary school children often solve additive word problems multiplicatively
- Younger children often solve multiplicative word problems additively
- Erroneous word problem solving occurs despite computation and discrimination skill
- Preference explains erroneous word problem solving on top of lacking skills

References

Bailey, D. H., Littlefield, A., & Geary, D. C. (2012). The codevelopment of skill at and preference for use of retrieval-based processes for solving addition problems: Individual and sex differences from first to sixth graders. *Journal of Experimental Child Psychology, 113*, 78-92. doi:10.1016/j.jecp.2012.04.014

Behr, M., & Harel, G. (1990). Understanding the multiplicative structure. In G. Booker, P. Cobb, & T.N. de Merldicutti (Eds.), *Proceedings of the 14th Conference of the International Group for the Psychology of Mathematics Education: Vol 3 (*pp. 27-34). Mexico, Mexico: Consejo Nacional de Ciencia y Technologia, Gobierno del Estado de Morelos.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects: State, trends, and issues in mathematics instruction. *Educational Studies in Mathematics, 22*, 37-68. doi:10.1007/BF00302716

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology, 44*, 1478–1490. doi:10.1037/a0013110

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education, 27*, 41-51. doi: 10.2307/749196

Cramer, K., & Post, T. (1993). Connecting research to teaching proportional reasoning, *Mathematics Teacher, 86*, 404-407.

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grades mathematics* (pp. 159-178). New York, NY: Macmillan.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (8th ed.). London, UK: Routledge.

Degrande, T. Verschaffel, L. & Van Dooren, W. (2017). Spontaneous Focusing on Quantitative Relations: Towards a Characterization, *Mathematical Thinking and Learning, 19*, 260-275, doi: 10.1080/10986065.2017.1365223

Degrande, T., Verschaffel, L., & Van Dooren, W. (2018). Beyond additive and multiplicative reasoning abilities: How preference enters the picture. *European Journal of Psychology of Education, 33,* 559-576. doi:10.1007/s10212-017-0352-y

Degrande, T., Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2018). Open word problems: Taking the additive or the multiplicative road? *ZDM Mathematics Education, 50,* 91-102, doi:10.1007/s11858-017-0900-6.

Deloof, G. (2005). *Leerlingvolgsysteem: Wiskunde – Toetsen 6 Basisboek* [Student monitoring system: Mathematics –Tests 6 Manual]. Antwerpen, Belgium: Garant.

Dudal P. (2002). *Leerlingvolgsysteem: Wiskunde – Toetsen 3 Basisboek* [Student monitoring system: Mathematics –Tests 3 Manual]. Leuven, Belgium: Garant.

Dudal P. (2003). *Leerlingvolgsysteem: Wiskunde – Toetsen 4 Basisboek* [Student monitoring system: Mathematics –Tests 4 Manual]. Antwerpen, Belgium: Garant.

Dudal, P., & Deloof, G. (2004). *Leerlingvolgsysteem: Wiskunde – Toetsen 5 Basisboek* [Student monitoring system: Mathematics –Tests 5 Manual]. Antwerpen, Belgium: Garant.

English, L. D. (1997). Promoting a problem-posing classroom. Teaching Children Mathematics, 4, 172–179.

English, L. D., & Lesh. R. A. (2003). Ends-in-view problems. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective on mathematics problem solving, learning, and teaching* (pp.297-316). Mahwah, NJ: Lawrence Erlbaum.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education*, *27,* 421-438. doi:10.1007/s10212-011-0087-0

Fischbein, R., Deri, M., Nello, M., & Marino, M. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*, 3-17. doi:10.2307/748969

Greer, B. (1992). Multiplication and division as models of situations. In D. Grouws (Ed.), *Handbook of research on learning and teaching mathematics* (pp. 276-295). Reston, VA: National Council of Teachers of Mathematics.

Greer, B., De Bock, D., & Van Dooren, W. (2009). The ISIS problem and pre-service teachers' ideas about proof. In F. Lin, F. Hsieh, G. Hanna & M. de Villiers (Eds), *ICMI Study 19 Conference Proceedings* (pp. 184-189). Taipei, Taiwan: National Taiwan Normal University.

Haciomeroglu, E. S., Chicken, E., & Dixon, J. K. (2013). Relationships between gender, cognitive ability, preference and calculus performance. *Mathematical Thinking and Learning*, *15*, 175-189, doi: 10.1080/10986065.2013.794255

Harel, G., & Behr, M. (1989). Structure and hierarchy of missing value proportion problems and their representations, *Journal of Mathematical Behavior*, *8*, 77-119.

Hart, K. (1981). Ratio and proportion. In K. Hart (Ed.), *Children's Understanding of Mathematics: 11- 16* (pp. 88-101). London, UK: John Murray.

Heller, P., Ahlgren, A., Post, T., Behr, M., & Lesh, R. (1989). Proportional Reasoning: The Effect of Two Context Variables, Rate Type and Problem Setting. *Journal for Research in Science Teaching*, 26, 205-220. doi: org/10.1002/tea.3660260303

Hoffer, A. (1988). Ratios and proportional thinking. In T. Post (Ed.), *Teaching mathematics in grades K-8: Research based methods* (pp. 285-313). Boston, MA: Allyn & Bacon.

Inglis, M. & Aberdein, A. (2015). Beauty is not simplicity: An analysis of mathematicians' proof appraisals. *Philosophia Mathematica, 23*, 87-109. doi: org/10.1093/philmat/nku014

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking for childhood to adolescence.* London, UK: Routledge.

Jeong, Y., Levine, S., & Huttenlocher, J. (2007). The development of proportional reasoning: Effect of continuous vs. discrete quantities. *Journal of Cognition and Development, 8*, 237–256. doi:10.1080/15248370701202471

Kaput, J. J., & West, M. M. (1994). Missing-value proportional reasoning problems: Factors affecting informal reasoning patterns. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 235-287). New York, NY: State University of New York Press.

Karplus, R., Pulos, S., & Stage, E. (1983). Proportional reasoning of early adolescents. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematical concepts and processes* (pp. 45–89). New York, NY: Academic Press.

Lamon, S. J. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for Research in Mathematics Education, 24,* 41-61. doi:10.2307/749385

Lamon, S. J. (2008). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (2nd ed.). New York, NY: Taylor & Francis Group.

Lamon, S. J., & Lesh, R. (1992). Interpreting responses to problems with several levels and types of correct answers. In R. Lesh & S. J. Lamon (Eds.), *Assessment of authentic performance in school mathematics* (pp. 319-342). Washington, DC: American Association for the Advancement of Science Press.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology, 124*, 83-97. doi:10.1037/0096-3445.124.1.83

Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Modestou, M., & Gagatsis, A. (2010). Cognitive and metacognitive aspects of proportional reasoning. Mathematical Thinking and Learning, 12, 36-53. doi: 10.1080/10986060903465822

Mulligan, J. (1992). Children's solutions to multiplication and division word problems: a longitudinal study. *Mathematics Education Research Journal, 4,* 24-41, doi:10.1007/BF03217230

Noelting, G. (1980). The development of proportional reasoning and the ratio concept: Part 1. Differentiation of stages. *Educational Studies in Mathematics, 11,* 217-253. doi:10.1007/BF00304357

Nunes, T., & Bryant, P. (2010). *Understanding relations and their graphical representation.* Retrieved from http://www.nuffieldfoundation.org/sites/defaukt/files/P4.pdf

Nunes, T., Bryant, P., Barros, R., & Sylva, K. (2012). The relative importance of two different mathematical abilities to mathematical achievement. *British Journal of Educational Psychology, 82*, 136–156. doi:10.1111/j.2044-8279.2011.02033.x.

Nunes, T., Bryant, P., Evans, D., & Barros, R. (2015). Assessing quantitative reasoning in young children, *Mathematical Thinking and Learning*, *17*, 178-196, doi:10.1080/10986065.2015.1016815

Pellegrino, J. W., & Glaser, R. (1982). Analyzing aptitudes for learning: Inductive reasoning. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 2, pp. 269-345). Hillsdale, NJ: Lawrence Erlbaum.

Resnick, L. B., & Singer, J. A. (1993). Protoquantitative origins of ratio reasoning. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 107-130). Hillsdale, NJ: Lawrence Erlbaum.

Schukajlow, S., Krug, A., & Rakoczy, K. (2015). Effects of prompting multiple solutions for modelling problems on students' performance. *Educational Studies in Mathematics, 89,* 393-417. doi:10.1007/s10649-015-9608-0

Siemon, D., Breed, M., & Virgona, J. (2005). From additive to multiplicative thinking: The big challenge of the middle years. In J. Mousley, L. Bragg, & C. Campbell (Eds.), *Proceedings of the 42nd Conference of the Mathematical Association of Victoria* (pp. 278-286). Melbourne, Australia: Mathematical Association of Victoria.

Silver, E. A. (1995). The nature and use of open problems in mathematics education: Mathematical and pedagogical perspectives. *ZDM/International Reviews on Mathematical Education, 2*, 67-72.

Thompson, P. W., & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick & G. Martin (Eds.), *Research companion to the NCTM Standards* (pp. 95-113). Washington, DC: National Council of Teachers of Mathematics.

Torbeyns, J., & Verschaffel, L. (2013). Efficient and flexible strategy use on multi-digit sums: a choice/no-choice study. *Research in Mathematics Education, 15*, 129-140. doi:10.1080/14794802.2013.797745

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics, 16,* 181 –204. doi: 10.1007/BF02400937

Van Dooren, W., Verschaffel, L., Greer, B., & De Bock, D. (2006). Modelling for life: Developing adaptive expertise in mathematical modelling from an early age. In L. Verschaffel, F. Dochy, M. Boekaerts & S. Vosniadou (Eds.), *Instructional psychology: Past, present and future trends. Sixteen essays in honour of Erik De Corte* (pp. 91–112). Oxford, UK: Elsevier.

Van Dooren, W., De Bock, D., Evers, M., & Verschaffel, L. (2009). Students' overuse of proportionality on missing-value problems: How numbers may change solutions. *Journal for Research in Mathematics Education, 40,* 187-211.

Van Dooren, W., De Bock, D., Vleugels, K., & Verschaffel, L. (2010). Just answering . . . or thinking? Contrasting pupils’ solutions and classifications of proportional and non-proportional word problems. *Mathematical Thinking and Learning, 12,* 20–35.

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication … and back. The development of students’ additive and multiplicative reasoning skills. *Cognition and Instruction, 28,* 360-381. doi: 10.1080/07370008.2010.488306

Van Dooren W., Vamvakoussi, X., & Verschaffel, L. (2018). Proportional reasoning. *Educational practice series, 30*, 1-34.

Van Dooren W., Verschaffel L., & Onghena P. (2002). The impact of preservice teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra word problems. *Journal for Research in Mathematics Education, 33,* 319-351. doi:10.2307/4149957

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 127-174). New York, NY: Academic Press.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 141-161). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse, The Netherlands: Swets & Zeitlinger.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557-628). Charlotte, NC: Information Age Publishing.